Design of IMC based PI Controller for Paper Machine Headbox

Parvesh Saini¹*, Rajesh Kumar²

¹Department of Electrical Engineering
²Department of Electronics Engineering
Graphic Era Deemed to be University, Dehradun, India
*Corresponding author: parvesh.saini.eee@geu.ac.in

(Received November 19, 2018; Accepted February 20, 2019)

Abstract
Headbox is one the major process of a paper machine which is vulnerable to many disturbances. Its efficient control is highly required because any disturbance in the process greatly affects the working of the headbox. Various tuning techniques have been designed and applied on paper machine headbox. This paper presents the design and analysis of Internal Model Control (IMC) based Multi Input – Multi Output (MIMO) - Proportional – Integral (PI) controller for paper machine headbox. Along with transient response, integral error performance indices (ISE, IAE, ITSE, and ITAE) have also been obtained. The performance of designed multivariable IMC – PI Controller has been compared with the conventional ZN - PI controller.

Keywords- Internal Model Control (IMC); Zeigler - Nichols (ZN); Paper Machine; Headbox; MIMO Controller.

1. Introduction
Centuries ago, the first paper came into existence in China (Ogunwusi and Ibrahim, 2014). Paper industry is an adding billion dollars to the world’s economy every year (Stewart et al., 2003) and hence, it plays an important role in the global economy. The economy of paper industry majorly depends on the quality of paper produced which in-turn depends on the efficient operation of the paper machine. A view of the paper machine has been shown in Figure 1.
The important subsystem of the paper machine is headbox (also known as flow box) which is used to spread pulp uniformly over the wire (Xiao and Wang, 2009). It is a highly nonlinear and complex two input - two output system with significant loop interaction. The headbox is subjected to disturbances from pumps, poor tuned controllers and variation in the concentration of the stock and there is strong loop interaction exists between the two loops (Nissinen et al., 1996). Hence, its precise control is highly required to cater to the need of better quality paper. For researchers, headbox has been an interesting process to design controller. In the past few decades, many control strategies have been developed for paper machine headbox. A brief review of such techniques has been discussed in (Saini and Kumar, 2018). Stock level and pressure inside the headbox are two major parameters which have to be controlled. Proper control of a system can be ensured only through its perfect mathematical modeling. For this paper, air cushioned pressurized headbox (Paattilammi and Makila, 2000) has been considered. A schematic of the headbox has been shown in Figure 2. The mathematical model of the headbox considered in the paper is explained in section 2.

![Figure 2. Headbox schematic](image)

The objective of this paper is to design a multivariable (MIMO) PI controller using IMC technique for paper machine headbox and to compare the performance with the conventional Zeigler - Nichols (ZN) technique. The following section gives a brief description of the mathematical model of the headbox.

2. Mathematical Model of Headbox
The headbox model considered for this work is given in (Nissinen et al., 1996; Paattilammi and Makila, 2000).
A 2x2 headbox model is given as:
However, the approximated model of headbox taking important dynamics into consideration is given as:

\[
\begin{bmatrix}
  y_1(s) \\
  y_2(s)
\end{bmatrix} = \begin{bmatrix}
  \frac{0.528 e^{-0.6s}}{2.2s+1} & \frac{0.205s + 0.00149}{43.6s^2 + s} \\
  \frac{(1.2539s + 0.63)}{30.051s^2 + 17.79s + 1} & \frac{(0.0007)}{s}
\end{bmatrix}
\begin{bmatrix}
  u_1(s) \\
  u_2(s)
\end{bmatrix} \quad (1)
\]

However, the approximated model of headbox taking important dynamics into consideration is given as:

\[
\begin{bmatrix}
  y_1(s) \\
  y_2(s)
\end{bmatrix} = \begin{bmatrix}
  \frac{0.528 e^{-0.6s}}{2.2s+1} & \frac{0.081}{1.89s + 1} \\
  \frac{1.49 \times 10^{-4} e^{-1.5s}}{s} & \frac{-7.0 \times 10^{-4} e^{-2s}}{s}
\end{bmatrix}
\begin{bmatrix}
  u_1(s) \\
  u_2(s)
\end{bmatrix} \quad (2)
\]

where \( y_1 \) and \( y_2 \) are pressure and stock level in the headbox respectively. And \( u_1 \) and \( u_2 \) are the feed pump speed and air valve position respectively. Here, pressure and stock level are the controlled variables. However, feed pump speed and air valve position are the manipulated variables. The process model as given by equation 1 and/or 3 includes four process elements which are given by:

\[
\begin{align*}
  g_{11} &= \frac{0.528 e^{-0.6s}}{2.2s+1}, &
  g_{12} &= \frac{0.081}{1.89s + 1}, \quad g_{21} = \frac{1.49 \times 10^{-4} e^{-1.5s}}{s}, &
  g_{22} &= \frac{-7.0 \times 10^{-4} e^{-2s}}{s},
\end{align*}
\]

where \( g_{11} \) is the transfer function between headbox pressure and feed pump speed, \( g_{12} \) is the transfer function between headbox pressure and air valve position, \( g_{21} \) is the transfer function between headbox stock level and feed pump speed, and \( g_{22} \) is the transfer function between headbox stock level and air valve position.

From above all transfer functions, “\( g_{11} \)” represents the headbox pressure loop and “\( g_{22} \)” represents the stock level loop of the headbox. However, “\( g_{12} \)” and “\( g_{21} \)” act as disturbances to headbox pressure and stock level respectively.

For a headbox, it is important to maintain the level of the stock and pressure inside it. These two parameters are greatly affected by the variations in the manipulated variables i.e. feed pump speed and air valve position. Air valve position act as disturbance parameter for the headbox pressure and feed pump speed act as a disturbance for the stock level. So, the variation in the above variables further affects the pressure on slice lip and jet velocity which in turn affect the rush - drag ratio. Ideally rush-drag ratio should be 1, however, practically its value must be as near to 1 as possible. So, to maintain the rush-drag ratio, feed pump speed and air valve position need to be manipulated precisely.
3. Internal Model Control (IMC) Based PI Controller Design
The integral model control (IMC) based PI/PID controller is broadly considered in industrial control. IMC method of controller tuning was developed by (Garcia and Morari, 1982). The design gives a superior concession along with set-point tracking, disturbance rejection, and robustness. The IMC design method is a kind of model-based control method based on the pole-zero cancellation (Saxena and Hote, 2016). A basic structure of IMC is shown in Figure 3. As shown, it consists of the original process (P) in parallel to a process model (PM). “Q” is the controller which receives the difference of original process and process model as feedback (Saxena and Hote, 2016). IMC is a simple technique based on the response of the actual plant (Y) and the response of the plant model (Y'). The input to IMC controller (Q) is the difference of Y and Y'. From the block diagram of IMC (Figure 3), the conventional controller QC can be obtained using the following expression:

\[
Q_c = \frac{Q}{1 - QPM}
\]  

From equation 4, it is clear that any IMC controller (Q) and Conventional Controller QC are equivalent to each other. Also, from the block diagram of the IMC control system (Figure 3), the closed-loop transfer function can be obtained as:

\[
Y = \frac{QP}{1 + Q(P - PM)}Y_{sp} + \frac{1 - QPM}{1 + Q(P - PM)}D
\]  

If, P = PM (ideal case), then equation 5 reduces to,

\[
Y = QPY_{sp} + (1 - QPM)D
\]  

Design of IMC includes two major steps.

**Step 1:** The process model is factored as

\[
PM = PM^*PM
\]  

where PM^* contains any time delays and right half plane zeros. Also, the steady state gain of PM^* must be unity so that the factors of equation 7 are unique.

**Step 2:** The controller is specified as

\[
Q(s) = \frac{1}{PM}f(s)
\]  

where “f(s)” is low pass filter having unity steady-state gain. The filter is generally represented by the form:
\[ f(s) = \frac{1}{(1 + \tau s)^r}, \quad r \in I, \tau_c > 0 \] (9)

Where “\( \tau \)” is the controller’s parameter and “\( \tau_c \)” is the design parameter to tune the controller for its robust performance. The value of “\( \tau_c \)” determines the speed of the response and is an important parameter in designing controller using IMC. Its selection is based on any of the three criteria mentioned below:

(a) \( \frac{\tau}{\theta} > 0.8 \) and \( \tau_c > 0.1\tau \) (Rivera et al., 1986)

(b) \( \tau > \tau_c > \theta \) (Chien and Fruehauf, 1990)

(c) \( \tau_c = \theta \) (Skogestad, 2003)

![Figure 3. Basic structure of IMC (Saxena and Hote, 2016)](image)

For this work, the plant’s transfer function as given by equation 2 has four types of process elements. In this model, \( g_{11} \) is a FOPDT process, \( g_{12} \) is a first order process, \( g_{21} \) & \( g_{22} \) are IPDT processes. Controllers have been designed for each process element by using the IMC – PI controller settings as explained in (Seborg et al., 2010). The tuning matrices of MIMO – PI controller are determined as follows:

\[
K_p = \begin{bmatrix}
    \frac{1}{k_{p11}} & \frac{1}{k_{p12}} \\
    \frac{1}{k_{p21}} & \frac{1}{k_{p22}}
\end{bmatrix}^{-1}
\] (10)

\[
K_I = \begin{bmatrix}
    \frac{1}{k_{i11}} & \frac{1}{k_{i12}} \\
    \frac{1}{k_{i21}} & \frac{1}{k_{i22}}
\end{bmatrix}^{-1}
\] (11)

The transfer function of MIMO – PI controller is given as:

\[
C(s) = K_p + K_I \frac{1}{s}
\] (12)

The matrix of the multivariable controller given by equation (12) is
\[ C(s) = \begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} \]  

(13)

where \( C_{11}(s) \) and \( C_{22}(s) \) are the main controllers for Pressure and Stock Level respectively and while, \( C_{12}(s) \) and \( C_{21}(s) \) are the cross - controllers for Stock Level and Pressure respectively.

4. Design and Result Analysis
The designed multivariable IMC - PI controller has been implemented using MATLAB/SIMULINK. The open loop Simulink model of headbox is shown in Figure 4. The open loop responses of headbox are shown in Figure 5. After evaluating the open loop performance of headbox, multivariable IMC-PI controller has been applied on headbox as depicted by Figure 6.

![Figure 4. Open loop simulink model of headbox](image)

![Figure 5. Open loop step response of headbox](image)
The controller parameters of the IMC based PI MIMO controller are given in Table 1 along with the value of “$\tau_c$”. The final value of “$\tau_c$” has been selected through some hit and trials on each processing element to get some optimal response of the given system.

![Figure 6. Simulink model of the headbox](image-url)

### Table 1. Value of $\tau_c$ and controller’s parameters

<table>
<thead>
<tr>
<th>Process</th>
<th>$\tau_c$</th>
<th>Controller's Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>g11</td>
<td>0.5</td>
<td>Kp: 3.78, Ti: 1.6</td>
</tr>
<tr>
<td>g12</td>
<td>0.5</td>
<td>Kp: 46.67, Ti: 1.89</td>
</tr>
<tr>
<td>g21</td>
<td>3.5</td>
<td>Kp: 850, Ti: 8.5</td>
</tr>
<tr>
<td>g22</td>
<td>3.5</td>
<td>Kp: -425.03, Ti: 9</td>
</tr>
</tbody>
</table>

Multivariable IMC – PI and ZN – PI controller gain matrices are given below:

$$Kp_{imc} = \begin{bmatrix} 3.64 & 33.15 \\ 1.82 & -408.45 \end{bmatrix}; \quad Ki_{imc} = \begin{bmatrix} 2.26 & 4.33 \\ 1.07 & -45.18 \end{bmatrix}; \quad Kp_{ZN} = \begin{bmatrix} 5.45 & 11.23 \\ 0.87 & -502.85 \end{bmatrix};$$

$$Ki_{ZN} = \begin{bmatrix} 3.006 & 0.21 \\ 0.36 & -75.63 \end{bmatrix}.$$

Using the parameters, as given in Table 1, multivariable IMC – PI controller has been designed and its performance on headbox has been evaluated. However, ZN – PI multivariable controller has been designed using conventional ZN tuning techniques. The transient response and performance indices have been determined and analyzed. The
respective values have been given in Table 2 and 3. The step responses of headbox for MIMO controller are shown from Figure 7 to Figure 10. In this, Figure 7 and Figure 8 represent the step responses of process “g11” and “g22” respectively. The processes “g12” and “g21” behave as disturbances for pressure and stock level loop respectively. Hence, Figure 9 and Figure 10 depict the disturbance rejection capability of PI - controller designed using IMC and ZN. From these figures it is observed that controllers show acceptable step responses and perfect disturbance rejection. Further, to evaluate the set point tracking of the controllers, MATLAB program is used. The set point tracking of the process “g11” and “g22” are indicated by Figure 11 and Figure 12 respectively. Similarly, disturbance rejection of controllers for set point changes have been depicted in Figure 13 and Figure 14 for the system “g21” and “g12” respectively. From the time response values (as depicted in Table 2), it is observed that IMC – PI multivariable controller gives better settling time and less overshoot as compared to ZN – PI multivariable controller. From the performance indices (as depicted in Table 3), it is observed that IMC – PI gives better performance than ZN – PI.

Table 2. Transient values of the system using IMC – PI controller

<table>
<thead>
<tr>
<th>Process</th>
<th>Tuning Technique</th>
<th>Rise Time (sec)</th>
<th>Overshoot</th>
<th>Settling Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (Y1 – U1)</td>
<td>IMC</td>
<td>0.90</td>
<td>17.32</td>
<td>4.27</td>
</tr>
<tr>
<td></td>
<td>ZN</td>
<td>0.60</td>
<td>35.41</td>
<td>5.78</td>
</tr>
<tr>
<td>Stock Level Y2 – U2</td>
<td>IMC</td>
<td>2.45</td>
<td>48.42</td>
<td>18.76</td>
</tr>
<tr>
<td></td>
<td>ZN</td>
<td>1.89</td>
<td>72.47</td>
<td>25.29</td>
</tr>
</tbody>
</table>

Table 3. Performance Indices

<table>
<thead>
<tr>
<th>Performance Indices</th>
<th>Pressure Loop</th>
<th>Stock Level Loop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IMC</td>
<td>ZN</td>
</tr>
<tr>
<td>ITSE</td>
<td>0.53</td>
<td>0.58</td>
</tr>
<tr>
<td>ITAE</td>
<td>1.66</td>
<td>1.85</td>
</tr>
<tr>
<td>IAE</td>
<td>1.37</td>
<td>1.44</td>
</tr>
<tr>
<td>ISE</td>
<td>0.91</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Figure 7. Step response headbox pressure (IMC (solid), ZN (dash))
Figure 8. Step response headbox stock level (IMC (solid), ZN (dash))

Figure 9. Step response of g12 (Disturbance) with IMC – PI controller (IMC (solid), ZN (dash))

Figure 10. Step response of g21 (disturbance) with IMC – PI controller (IMC (solid), ZN (dash))
Figure 11. Set point tracking of headbox pressure (g11)

Figure 12. Set point tracking of headbox stock level (g22)

Figure 13. Disturbance rejection with set-point changes (stock level v/s feed pump speed)
5. Conclusion
This paper has presented the design and analysis of IMC based multivariable PI controller for paper machine headbox and its comparison with the performance of multivariable ZN – PI controller. From the analysis of the system, it has been observed that IMC – PI controller has given a moderate transient response with acceptable disturbance rejection and performance indices.

References


